Data Structures & Algorithms @ AC2025

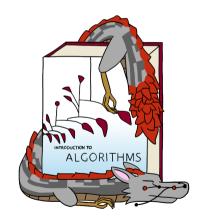
Deep C Adventures

Bunsen Bitti, Unsigned Long

July 3, 2025

Overview

- This is a comedic/informative look at overlaps between furry and computer science
 - This is not a rigorous lecture
 - There will be puns ^w^



Overview

- This is a comedic/informative look at overlaps between furry and computer science
 - This is not a rigorous lecture
 - There will be puns ^w^
- Don't worry if our jokes don't make sense!
 - We've been coding for most of our lives
 - If you're confused, that's on us
 - We're out of touch with reality



Found furries through con videos



- Found furries through con videos
- Found some eastern dragons with the word "Long" in their name



Tien Long

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- Thought about "long" for too long



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- Realized it's in my code

short short int signed short signed short int	Short signed integer type. Capable of containing at least the [-32 767, +32 767] range [3][a]
unsigned short unsigned short int	Short unsigned integer type. Contains at least the [0, 65 535] range. [3]
int signed signed int	Basic signed integer type. Capable of containing at least the [-32 767, +32 767] range. [3][a]
unsigned unsigned int	Basic unsigned integer type. Contains at least the [0, 65 535] range. ^[3]
long long int signed long signed long int	Long signed integer type. Capable of containing at least the [-2 147 483 647, +2 147 483 647] range. [3][8]
unsigned long unsigned long int	Long unsigned integer type. Capable of containing at least the [0, 4 294 967 295] range. [3]

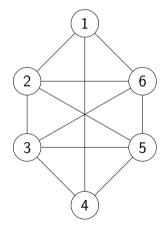
- Found furries through con videos
- Found some eastern dragons with the word "Long" in their name
- Thought about "long" for too long
- Realized it's in my code
- Pun was too good to do nothing with, so I made a fursona





Fursuits are Planar Graphs

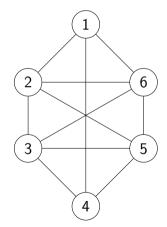
- Graph:
 - Vertices labeled 1, 2, . . . , n
 - Edges go between two vertices
 - At most $n \cdot (n-1)/2$ edges^a



^aAssuming simple graphs

Fursuits are Planar Graphs

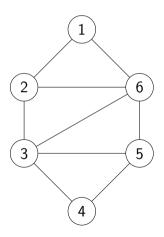
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Fursuits are Planar Graphs

- Graph:
 - Vertices labeled 1, 2, . . . , n
 - Edges go between two vertices
 - At most $n \cdot (n-1)/2 = O(n^2)$ edges^a
- Planar graphs: graphs that can be drawn without crossing edges
 - At most 3n 6 = O(n) edges!^b
 - Fursuits are planar graphs
 - Seam count = O(Fabric patch count)

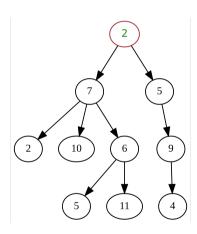


^aAssuming simple graphs

^bAssuming faces have degree > 3

Trees!

- Trees are the building block of most data structures.
- Every node has one parent, except for the root which has no parent.
- Nodes with no children are leaves, nodes with children are internal nodes.



By Paddy3118 - Own work, CC BY-SA 4.0,

https://commons.wikimedia.org/w/index.php?curid=83223854

A-Z of Trees (non-exhaustive)

AVL Trees k-d Tree

B Trees Link-Cut Tree

Cartesian Trees Merkle Trees Universal B Trees

Decision Tree N Tree Vantage Point Tree

Exponential Trees Order Statistics Tree Wallace Tree

Fenwick Trees PQ Tree X-fast Trie

Gomory-Hu Tree Quadtree Y-fast Trie

H-Tree Red Black Trees Zip Trees

Interval Tree Splay Trees

Judy Array Tango Trees

Subsections Subtrees

- ARSTZ : Balanced Binary Search Trees (BST)
- BCMUXY : Other Binary Trees
- IKQV : Spatial Data Structures
- EFGJLP : Niche data structures
- DHNOW : Non-data structures

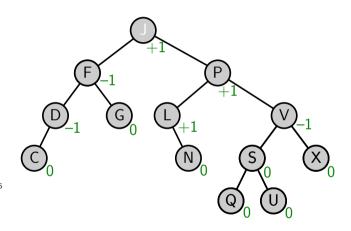
Subsections Subtrees

- ARSTZ : Balanced Binary Search Trees (BST)
- BCMUXY : Other Binary Trees
- IKQV : Spatial Data Structures
- **E**FGJ**L**P : Niche data structures
- **DHN**OW : Non-data structures

AVL Trees¹

- First $O(\log n)$ self-balancing binary search tree!
- ullet Ensures the height of its children do not differ by >1

 ${\tt https://commons.wikimedia.org/w/index.php?curid=} 49182185$



¹Adelson-Velsky, Georgy; Landis, Evgenii (1962)

AVL Trees (Rotation)

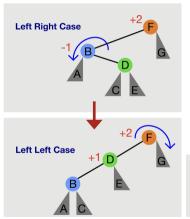
 Imbalances are fixed by rotations, fundamental local operations for many balanced BSTs

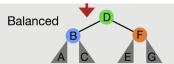


/r/furry_irl/comments/1dumfg3/

AVL Trees (Rotation)

 Imbalances are fixed by rotations, fundamental local operations for many balanced BSTs

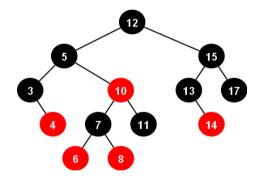




https://pages.cs.wisc.edu/~qingyi/

Red Black Tree²

- Balanced BST, red-black coloring chosen because it looked best on the laser printer they used.
- All root-to-leaf paths have the same number of black nodes
- No red node has a red parent



https://pages.cs.wisc.edu/~wyoungjun/

²Guibas, Leonidas J.; Sedgewick, Robert (1978).

Splay?

```
splay 1 of 3 verb

splayed; splaying; splays

Synonyms of splay >

transitive verb

1 : to cause to spread outward

2 : to make oblique: BEVEL

intransitive verb
```



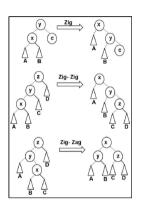
https://en.wikifur.com/wiki/Furpile

: SLOPE, SLANT

: to extend apart or outward especially in an awkward manner

Splay Tree³

- After every operation, splay the searched node to the root of the tree using double-rotations.
- Suspected to be optimal for any sequence of BST operations up to a constant factor (Dynamic Optimality Conjecture)



Trabelsi, Zouheir & Zeidan, Safaa & Masud, Mehedy & Ghoudi, Kilani.

(2015). Statistical Dynamic Splay Tree Filters.

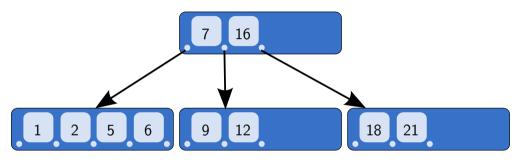
³Sleator, Daniel D.; Tarjan, Robert E. (1985).

B Tree⁴

- Generalising binary search trees to > 2 children
- Internal nodes hold both data and pointers to children nodes in sorted order
- Forms the basis of many filesystem structures
 - Apple HFS+, Microsoft NTFS, Linux ext4

⁴Bayer, R.; McCreight, E. (July 1970).

B Tree (Diagram)



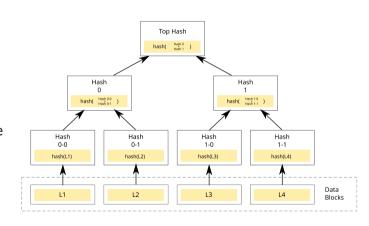
By CyHawk, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=11701365

What Rudy [Bayer] likes to say is, the more you think about what the B in B-Tree means, the better you understand B-Trees!

- Edward M. McCreight

Merkle Tree⁵

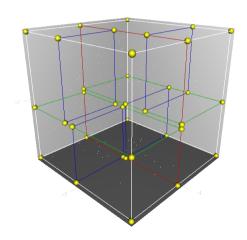
- Tree of hashes, widely used for data verification
 - Cryptographic schemes
 - P2P file sharing
 - Distrbuted filesystems
- Each internal node hashes the concatenated hashes of their children.
- Only recomputes O(log n) hashes when data changes!



⁵Merkle, R. C. (1979)

k-d tree⁶

- Each node is a k-dimensional point, each internal node splits one of the dimensions at the point.
- O(log n) insert, delete and search for a point, fast^a range search and nearest neighbour search.



^asubject to curse of dimensionality

⁶Bentley, J. L. (1975).

Exponential Trees⁷

- Has niche uses in hash tables
- Time complexity is a bit cursed

Abstract

We present a significant improvement on linear space deterministic sorting and searching. On a unit-cost RAM with word size w, an ordered set of n w-bit keys (viewed as binary strings or integers) can be maintained in

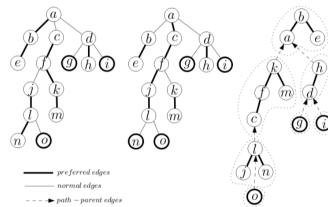
$$O\left(\min\left\{\begin{array}{l} \sqrt{\log n} \\ \frac{\log n}{\log w} + \log\log n \\ \log w \log\log n \end{array}\right\}\right)$$

⁷Andersson, Arne (October 1996)



Link-Cut Tree⁸

- Maintains a set of rooted trees
- Amortized O(log n) link, cut and find-root at any node
- Improves Dinic's Algorithm (for max-flow) from $O(V^2E)$ to $O(VE \log V)$.

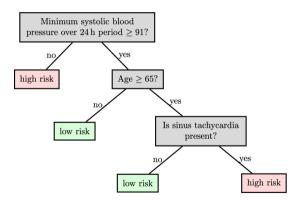


By Drrilll, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=25495327

⁸Sleator, D. D.; Tarjan, R. E. (1983).

Decision Trees

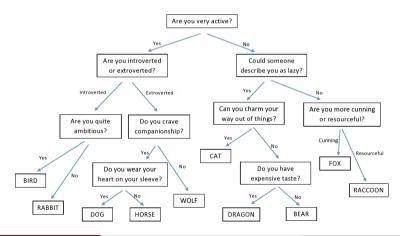
- Each node represents a query, each leaf represents a category/decision.
- Commonly used in decision analysis and machine learning
- Fuzzy Decision Trees are also a thing



MIT 6.390 Intro ML Course Notes and Breiman, Friedman, Olshen, Stone (1984)

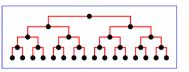
(Furry) Decision Trees (r/furry/comments/4gxm01/)

What is your fursona?



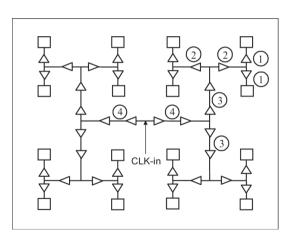
H Trees

- Binary tree fractal!
- Used in chip design in order to distribute wide reaching nets





https://www.tamurajones.net/FractalGenealogy.xhtml



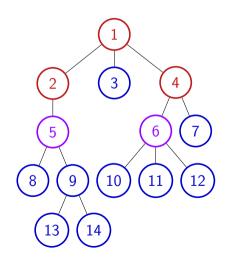
Tawfik, Sherif & Kursun, Volkan. (2008).

Missing N-tree

Seriously I could not find any tree data structure that starts with N

Macro-Micro Tree Decomposition

- Node is micro if it has less than O(log(n)) descendants (else macro)
- Node is macro leaf if it is macro and all its children are micro
- Subtree is microtree if its parent is a macro leaf
- There are at most $O(n/\log(n))$ macro leaves
 - Macro leaves have O(log(n)) decendants, and do not share decendants
- There are $O(n^{1/c})$ distinct microtree shapes
 - Tree shape count is exponential in node count
 - Number of microtree nodes is logarithmic in *n*
 - log and exp cancel out to $O(n^{1/c})$



Fast Furrier Transform

- Fourier Transform?
 - Typically computed with complex numbers
 - Transform signal between time and frequency domain

Fast Furrier Transform

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 - Typically computed with complex numbers
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- This is not a math panel!
 - See this 3Blue1Brown video:

"But what is the Fourier Transform? A visual introduction." \rightarrow



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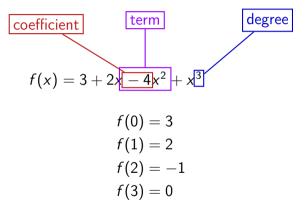
"But what is the Fourier Transform? A visual introduction." ightarrow

- This is an algorithms panel :3
 - We will use FFT to multiply polynomials in $O(n \log n)$ time

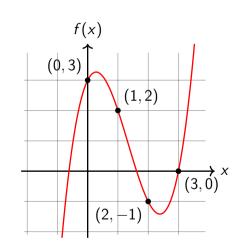


Data Structures & Algorithms @ AC2025

Fast Furrier Transform: Polynomials



Evaluating a polynomial at a point takes O(n) time



Fast Furrier Transform: Multiplication

$$f(x) = 3 + 2x - 4x^{2} + x^{3}$$

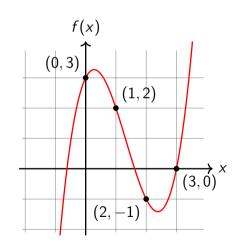
$$g(x) = 2 - x + x^{2}$$

$$h(x) = f(x) \cdot g(x) = 6 + x - 7x^2 + 8x^3 - 5x^4 + x^5$$

Multiplying two polynomials of degree n by distributing takes $O(n^2)$ time

Fast Furrier Transform: Polynomial Interpolation/Evaluation

- For a set of n points, there is a unique polynomial with degree less than n that passes through all the points
- Evaluating the polynomial at n different x values to find these points typically takes O(n²) time
- If we choose the x values cleverly, we can do better



$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_{n-2}x^{n-2} + a_{n-1}x^{n-1}$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{n-2}x^{n-2} + a_{n-1}x^{n-1}$$

$$f(1) = a_0 + a_1 + a_2 + a_3 + \dots + a_{n-2} + a_{n-1}$$

$$f(-1) = a_0 - a_1 + a_2 - a_3 + \dots + a_{n-2} - a_{n-1}$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{n-2}x^{n-2} + a_{n-1}x^{n-1}$$

$$f(1) = a_0 + a_1 + a_2 + a_3 + \cdots + a_{n-2} + a_{n-1}$$

 $f(-1) = a_0 - a_1 + a_2 - a_3 + \cdots + a_{n-2} - a_{n-1}$

$$f(1) = (a_0 + a_2 + \cdots + a_{n-2}) + (a_1 + a_3 + \cdots + a_{n-1})$$

 $f(-1) = (a_0 + a_2 + \cdots + a_{n-2}) - (a_1 + a_3 + \cdots + a_{n-1})$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{n-2} x^{n-2} + a_{n-1} x^{n-1}$$

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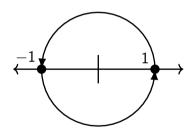
$$f(1) = (a_0 + a_2 + \cdots + a_{n-2}) + (a_1 + a_3 + \cdots + a_{n-1})$$

 $f(-1) = (a_0 + a_2 + \cdots + a_{n-2}) - (a_1 + a_3 + \cdots + a_{n-1})$

Number of operations is halved!

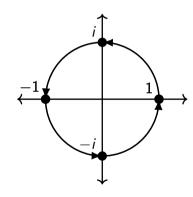
Fast Furrier Transform: Roots of Unity

- ullet The even/odd trick works because repeatedly multiplying by -1 cycles between -1 and 1
- It'd be convenient if we had other values making this kind of cycle, and if these cycles were longer



Fast Furrier Transform: Roots of Unity

- ullet The even/odd trick works because repeatedly multiplying by -1 cycles between -1 and 1
- It'd be convenient if we had other values making this kind of cycle, and if these cycles were longer
- Complex numbers give us values r where $r^n = 1$
- These are called roots of unity
- FFT evaluates f at $1, r, r^2, \ldots, r^{n-1}$



$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1}$$

$$f_e(x) = a_0 + a_2 x^2 + a_4 x^4 + \dots + a_{n-2} x^{n-2}$$

 $f_2(x) = a_1 x + a_3 x^3 + a_5 x^5 + \dots + a_{n-1} x^{n-1}$

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$$

$$f_e(x) = a_0 + a_2 x^2 + a_4 x^4 + \dots + a_{n-2} x^{n-2}$$

 $f_o(x) = a_1 x + a_3 x^3 + a_5 x^5 + \dots + a_{n-1} x^{n-1}$

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$$f_o(x) = a_1 x + a_3 x^3 + a_5 x^5 + \dots + a_{n-1} x^{n-1}$$

$$f_o(x) = x(a_1 + a_3 x^2 + a_5 x^4 + \dots + a_{n-1} x^{n-2})$$

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$$g_e(y) = a_0 + a_2 y + a_4 y^2 + \dots + a_{n-2} y^{n/2-1}$$

$$g_o(y) = a_1 + a_3 y + a_5 y^2 + \dots + a_{n-1} y^{n/2-1}$$

$$f(x) = g_e(x^2) + xg_o(x^2)$$

Fast Furrier Transform: Divide and Conquer

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1} = g_e(x^2) + x g_o(x^2)$$

$$g_e(y) = a_0 + a_2 y + a_4 y^2 + \dots + a_{n-2} y^{n/2-1}$$

$$g_o(y) = a_1 + a_3 y + a_5 y^2 + \dots + a_{n-1} y^{n/2-1}$$

• We need to evaluate g_e and g_o at $y = x^2 =$

$$(1)^2$$
, $(r)^2$, $(r^2)^2$,..., $(r^{n/2-1})^2$, $(r^{n/2})^2$, $(r^{n/2+1})^2$,..., $(r^{n-1})^2$

Fast Furrier Transform: Divide and Conquer

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$$(1)^{2}, \quad (r)^{2}, \quad (r^{2})^{2}, \dots, \quad (r^{n/2-1})^{2}, \quad (r^{n/2})^{2}, \quad (r^{n/2+1})^{2}, \dots, \quad (r^{n-1})^{2}$$

$$1, \quad r^{2}, \quad r^{4}, \dots, \quad r^{n-2}, \quad r^{n} = 1, \qquad r^{2}, \dots, \quad r^{n-2}$$

Fast Furrier Transform: Divide and Conquer

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1} = g_e(x^2) + x g_o(x^2)$$

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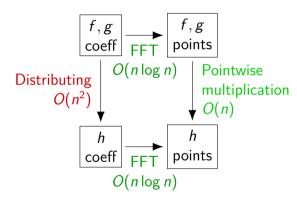
$$(1)^{2}, \quad (r)^{2}, \quad (r^{2})^{2}, \dots, \quad (r^{n/2-1})^{2}, \quad (r^{n/2})^{2}, \quad (r^{n/2+1})^{2}, \dots, \quad (r^{n-1})^{2}$$

$$1, \quad r^{2}, \quad r^{4}, \dots, \quad r^{n-2}, \quad r^{n} = 1, \qquad r^{2}, \dots, \quad r^{n-2}$$

- To evaluate f at n roots of unity, we evaluate g_e and g_o at n/2 roots of unity
- Recursion! $T(n) = 2T(n/2) + O(n) = O(n \log n)$

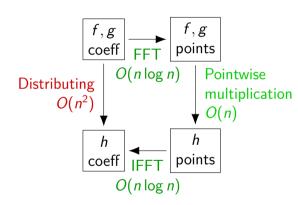
Fast Furrier Transform: Inverse FFT?

- Multiplying polynomials of degree n
 - Distributing would take $O(n^2)$
 - FFT takes $O(n \log n)$
 - Pointwise multiplication takes O(n)

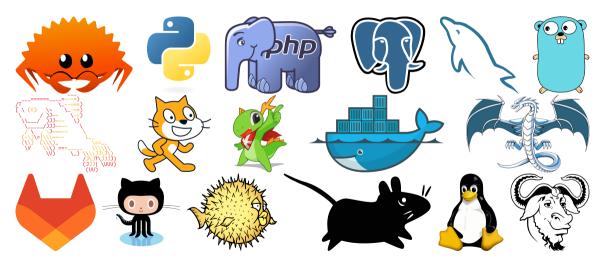


Fast Furrier Transform: Inverse FFT?

- Multiplying polynomials of degree n
 - Distributing would take $O(n^2)$
 - FFT takes $O(n \log n)$
 - Pointwise multiplication takes O(n)
- FFT is (approximately) its own inverse
 - No proof here: not a math panel
 - IFFT takes $O(n \log n)$
 - The Anti-Furry Transform is just a Furry Transform in disguise :3



Animal Computing Mascots



Animal Computing Mascots: Trans Rights!



Animal Computing Mascots: Unofficial Mascot of C++



Animal Computing Mascots: Powershell...



Animal Computing Mascots: WSL

From: Richard Stallman

Subject: WSL

Date: Mon, 23 Jan 2023 22:50:01 -0500

```
[[[ To any NSA and FBI agents reading my email: please consider ]]]
[[[ whether defending the US Constitution against all enemies, ]]]
[[[ foreign or domestic, requires you to follow Snowden's example. ]]]
```

How about pronouncing (and writing) "WSL" as "weasel"?

--

Dr Richard Stallman (https://stallman.org)
Chief GNUisance of the GNU Project (https://gnu.org)
Founder, Free Software Foundation (https://fsf.org)
Internet Hall-of-Famer (https://internethalloffame.org)



Gray Himakar

- Normal counting can have many digits change when going to the next value
 - Physical switches changing might not be simultaneous

Binary	
000	
001	
010	
011	
100	
101	
110	
111	

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- Binary reflected gray code (BRGC): to add a bit, reflect existing sequence, give one half a 0, and the other half a 1

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010	11
011	10
100	10
101	11
110	01
111	00

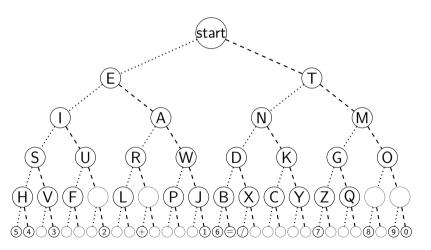
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001	001
010	011
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100	1 10
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111	1 00

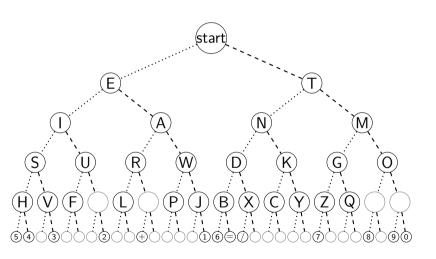
- Normal counting can have many digits change when going to the next value
 - Physical switches changing might not be simultaneous
- Binary reflected gray code (BRGC): to add a bit, reflect existing sequence, give one half a 0, and the other half a 1
- There are cool bit hacks for computing the next BRGC value and converting between binary and BRGC
- There are also variants that balance the number of transitions per bit

Binary	BRGC
000	000
001	001
010	011
011	010
100	110
101	111
110	101
111	100

Morse Code

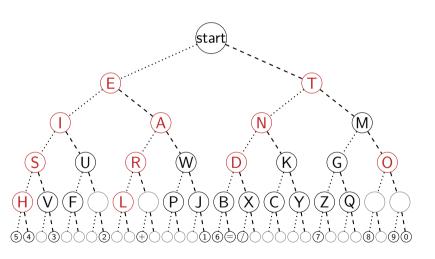


Morse Code



Letter	Frequency
Е	12.7%
Т	9.1%
Α	8.2%
Ο	7.5%
I	7.0%
N	6.7%
S	6.3%
Н	6.1%
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Aroga	1	Unified	English	Braille	Chart				
ALPHABET AND NUMBERS	PUNCTUATION	SIGNS OF OPERATION AND COMPARISON	ALPHABETIC WORDSIGNS	STRONG GROUPSIGNS	INITIAL-LETTER CONTRACTIONS	FINAL-LETTER GROUPSIGNS	SHORTFORM WORDS		
1 2 3 4 5 6 7 8 9 0	· comma		b but	* di	day	GROUPSIGNS	ab about	hggf hervelf	
abedefghij	period .	plus +	c can	: a		ound	aby above ac according	hm him hmf himadf	
	apostrophe	minus -	d do		• • ever	ance	acr across	imm immediate	
k i m n o p q r s t	ecton :	• : multiplication x	e every f from	th	father	sion	af after afn afternoon	as its af itself	
u v x v z w	duh.	· · · multiplication dot ·	g go	wh	here	less	afw afterward	le letter	
111111111111111111111111111111111111111		division +	h have	ou	know	ount	ag again	II little	
	in the long dash —	fire greater than >	j just	88	lord		agair against alm almost	m <u>ch</u> much mgt must	
INDICATORS	: exclamation mark	i less than <	k knowledge	gh	mother mother	to the ence	alr already	myi myself	
Numeric	hyphen -		l like m more	ed ed	name		akh although	nec necessary nei neither	
Capital	: question mark ?	equals -	n not	er	one	ong	alt altogether	cast oncodi	
letter	semicolon ;	CURRENCY AND MEASUREMENT	p people	ow		ful	also always hee because	gares ourselves pd paid	
word	TTT t ellipsis		q quite			tion	hel before beh behind	pggcv perceive	
passage	forward slash /	cent 4	r rather	ar ar		ness	bel below	pggcvg perceiving pgh perhaps	
capital terminator	backward slash \	dollar S	s so t that	ing ing	right	ment	han beneath	qk quick	
Grade 1	opening outer	* * • euro €	u us	LOWER	some	ity	her beside het between	rcv receive rcvg receiving	
symbol	i guotation mark	: * : British Pound 6	v very	GROUPSIGNS	time		lagy buyond	rje rejoice	
	closing outer	feet '	w will	• ea	under		bl blind bel braille	rjeg rejoicing rd said	
: word	at quotation mark	inches "	x it	t bb	work		chin children	ahd should	
I I I passage	· · · · opening inner		y you z as	- 60	young		conceive conceiving	ngh such therry themselves	
Grade 1 terminator	quotation mark	SPECIAL SYMBOLS		ii n	:: at young		ed could	thyf thyself	
Typeform	closing inner	percent %	STRONG CONTRACTIONS (Part and Whole Word)				dev deceive deve deceiving	td today ter together	
italic symbol	quotation mark	degree			there		del declare	tm tomorrow	
italic word		angle 4	and	t be	character		delg declaring ei either	tn tonight well would	
italic passage		hashtag #	for	ee con	through		fat first	yr your	
0 • 01 man	GROUPING PUNCTUATION		ef of	dis	where		fr friend gd good	yrf yourself yrrs yourselves	
	• ‡ opening round	· · · · · · · · · · · · · · · · · · ·	the	en	ought		grt great		
	parenthesis (with	in in	ween		Retired Contractions		
bold word	closing round	trademark	STRONG WORDSIGNS		word		(not used in UEB)		
bold passage	parenthesis) opening square	superscript		LOWER WORDSIGNS	these	: ble	ation	ally	
bold terminator	bracket	subscript	: child	: be				1.11	
underline symbol	closing square	indicator	shall	enough		÷÷ dd	com	to to	
underline word	bracket]	bullet •	this	:: were	whose				
underline word underline passage	opening curly bracket	e sign e	which	. his		into into	by	o'c o'clock	
	closing curly	asterisk .	out	in in	cannot				
underline terminator	bracket)	* ## det locator for mention	still	was	had		brought to you by		
script symbol	opening angle				many		ATECHNOLOGIE	ES	
script word	: : closing angle				spirit		Aroga		
script passage	bracket >				world		uuga	i.i.	
script terminator						Visit our online store at www.aroga.com			
	'		'		their		© Aroga Technologies 2014		

ALPHABET AND NUMBERS

```
1 2 3 4 5 6 7 8 9 0 a b c d e f g h i j k l m n o p q r s t u v x y z w
```

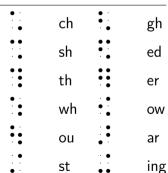
ALPHABET AND NUMBERS

a b c d e f g h i j

k l m n o p q r s t

u v x y z w

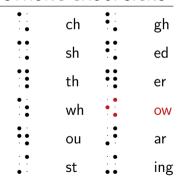
STRONG GROUPSIGNS



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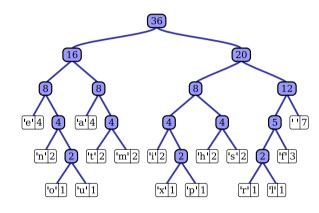
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"owo" is

Huffman Coding

- More frequent characters get shorter encodings
- Used in .zip files and other compression algorithms
- Won't go over details due to time

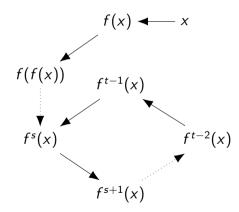


Tortoise and Hare: The Cycle Finding Problem

- Take some function $f : [n] \rightarrow [n]$, and some starting value x.
 - Notation: [n] is the numbers $\{1,\ldots,n\}$, and f is a function that takes a number from [n], and outputs a number in [n]

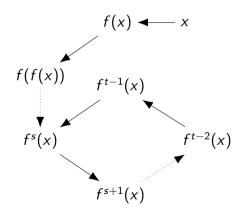
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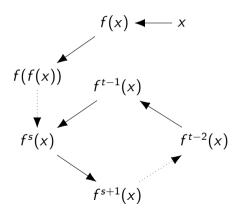
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- Find s, t such that $f^s(x) = f^{s+t}(x)$



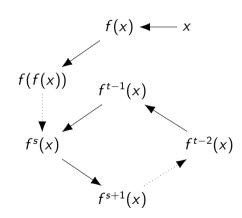
Tortoise and Hare: Floyd's Algorithm

- Algorithm description
 - Tortoise and hare start with $x_t := x$ and $x_h := x$.
 - When tortoise computes $x_t := f(x_t)$, hare computes $x_h := f(f(x_h))$.



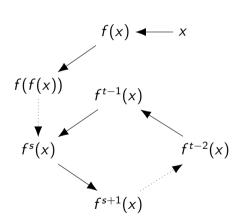
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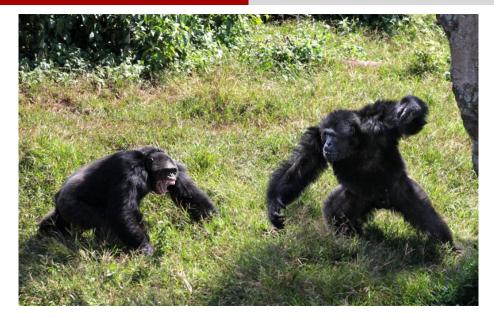
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- Used in Pollard's rho algorithm
 - Pollard's kangaroo exists???





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- There is a transformation that moves primal constraints into objective function terms, and moves primal objective function terms into constraints
 - The resulting formulation is called the "dual"
- Some optimization algorithms switch between primal and dual formulations
 - Primal-dual methods

Data Structures & Algorithms @ AC2025 Deep C Adventures

Bunsen Bitti, Unsigned Long

July 3, 2025