Welcome to Data Structures and Algorithms! I'm InverseHackermann, and I will be mainly talking about dynamic programming. If you get bored of me talking, here are some problems to work on! They are (subjectively) ordered by difficulty, and we will go over the first three together. Feel free to ask questions throughout the panel!

## Problem 1:

Define the Fibonacci numbers as fib(0) = 0, fib(1) = 1, and fib(n) = fib(n-1) + fib(n-2). Given  $n \le 10^6$ , what is the *n*th Fibonacci number? Since the answer may be large, return the result mod  $10^9 + 7$ .

What if  $n < 10^{18}$ ?

## Problem 2:

Given  $n \le 100$ , a list of n integers  $a_1, \ldots, a_n \le 100$ , and an integer k, is it possible to pick some of the numbers such that their sum is k?

#### Problem 3:

Define an modification to a string as an insertion of a character, deletion of a character, or substitution of one character for another. Given two strings s and t, where  $|s|, |t| \le 10^3$ , what is the minimum number of modifications to transform s into t?

### Problem 4:

Consider a game starting with a line of  $n \leq 100$  boxes of various colors. In a single move, you can remove a contiguous range of k boxes that have the same color, and earn  $k^2$  points. What is the maximum number of points you can get?

### Problem 5:

Given  $n \leq 10^5$ , what is the number of n digit numbers that are not divisible by 7 and do not have equal consecutive digits? Since the answer may be large, return the result mod  $10^9 + 7$ .

What if  $n \leq 10^{18}$ ? What about numbers not divisible by 35? Or numbers not divisible by 175?

## Problem 6:

Given a sequence of  $n \leq 100$  numbers and an integer  $k \leq 10$ , what is the number of length-k increasing subsequences? Subsequences are obtained by deleting any amount of numbers from the original sequence.

What if  $n \leq 10^4$ ?

## Problem 7:

There are  $n \leq 10^3$  fursuiters in a line, each with a species  $s_i$  and a height  $h_i$ . They want their photos taken in groups, where each fursuiter appears in exactly one group, and groups are formed from contiguous sequences of fursuiters. Furthermore, for the sake of diversity, a group should not contain two fursuiters of the same species. The cost to take a photo of a group is the height of the tallest fursuiter in the group, and the overall cost is the sum of the costs of all photos. What is the minimum cost to take the photos of the fursuiters?

What if  $n < 10^5$ ?

# Problem 8:

Given a permutation of the numbers  $1, \ldots, n$ , define the "max/min sequence" to be generated as follows: Insert the numbers into a set one at a time. For all numbers except the first, if the inserted number is the new minimum, add "<" to the sequence. Otherwise, if it is the new maximum, add ">". Otherwise, add "?".

Given a particular sequence of characters s of length  $|s| \le 10^3$  formed from <, >, and ?, how many permutations have this sequence as their max/min sequence? Since the answer may be large, return the result mod  $10^9 + 7$ .

What if  $|s| < 10^5$ ? What if I want the result after each of  $m < 10^5$  character substitutions of s?